CATEGORY THEORY

IVAN DI LIBERTI

EXERCISES

Leinster (🪫). 0.13

Leinster (🪫). 0.14

Leinster (🪫). 1.1.13

Leinster (🪫). 1.1.15

Leinster (🔋). 1.3.27

Leinster (🔋). 1.3.32

Riehl (🪫). 1.1.iii

Riehl (🪫). 1.3.vi

Leinster (🔋). Prove that there is no functor $\mathsf{Grp} \to \mathsf{Ab}$ sending each group G to its center.

Leinster (🔋). 1.2.24

Leinster (🪫). 1.3.28. Moreover, show that the assignment $X \mapsto A \times X^A$ is functorial and that (a) provides a natural transformation $A \times (-)^A \Rightarrow (-)$. Provide a similar statement for (b).

Leinster (🔋). 1.3.31

Exercise 1 (☕, 🔋, G-sets). A G-set is a couple (X, χ_X) where X is a set and $\chi : G \times X \to X$ is an action. A morphism of G-sets $f : (X, \chi_X) \to (Y, \chi_Y)$ is a G-equivariant set function. Show that the category of G-sets is equivalent to the category of functors (and natural transformations between them) $\mathbf{Set}^G$, where G is regarded as a 1-object category.

Exercise 2 (☕, 🔋, Quiv). Let $(\cdot \rightrightarrows \cdot)$ be a category with two objects $\{a, b\}$, identities, and two distinct maps $f, g : a \rightrightarrows b$. Let Quiv be the category of quivers and morphisms between them. Show that:

$$\mathrm{Quiv} \simeq \mathbf{Set}^{(\cdot \rightrightarrows \cdot)}.$$

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Exercise 3 (🔋). Show that $\mathcal{K}$ has a faithful and conservative functor $U : \mathcal{K} \to \mathbf{Set}$ if and only if it has a faithful functor $V : \mathcal{K} \to \mathbf{Set}$ and a conservative functor $Z : \mathcal{K} \to \mathbf{Set}$.

Exercise 4 (🔋). Show that if $\mathcal{K}$ has a faithful functor $\mathcal{K} \to \mathbf{Set}$, so does its opposite category $\mathcal{K}^{\circ}$.

Exercise 5 (🔋). Show that every small category C has a faithful functor $C \to \mathbf{Set}$. Why the same argument does not work for a locally small category?

The riddle of the week (☕, ⚠). Show that every locally finite category $\mathcal{K}$ has a conservative functor into the category of finite sets $\mathcal{K} \to \mathsf{Fin}$.

● (red) the exercises in the red group are mandatory.
● (yellow) pick at least one exercise from each of the yellow groups.
● (blue) pick at least two exercises from each of the blue groups.
● (brown) nothing is mandatory in the brown box.
● (green) The riddle of the week. It's just there to let you think about it. It is not a mandatory exercise, nor it counts for your evaluation. Yet, it has a lot to teach.
☕ useful to deepen your understanding. Take your time to solve it. (May not be challenging at all.)
🔋 measures the difficulty of the exercise. Note that a technically easy exercise is still very important for the foundations of your knowledge.
⚠ It's just too hard.
The label **Leinster** refers to the book **Basic Category Theory**, by *Leinster*.
The label **Riehl** refers to the book **Category Theory in context**, by *Riehl*.