CATEGORY THEORY 2025

IVAN DI LIBERTI

rules

- Hand your exercises by the **6th lecture** via email. In order to make my life easier, make sure to include the word **CT25 in the subject**.
- Pick at least one exercise from each of the yellow groups.
- You must charge at least **2** batteries and a half!
  Example. The vector of exercises [3,6,8,9,14,20] would pass this sheet.

EXERCISES

categories, functors, natural transformations

Borceux 1 (🪫). 1.11.2

Borceux 2 (🪫). 1.11.3

Borceux 3 (🪫). 1.11.4

Borceux 4 (🪫). 1.11.9

Leinster 5 (🪫). 1.2.24

Exercise 6 (☕, 🪫). Let $(\cdot \rightrightarrows \cdot)$ be a category with two objects $\{a, b\}$, identities, and two distinct maps $f, g : a \rightrightarrows b$. Let Quiv be the category of quivers and morphisms between them. Show that:

$$\text{Quiv} \simeq \mathbf{Set}^{(\cdot \rightrightarrows \cdot)}.$$

(co)limits

Borceux 7 (🪫). 2.17.1

Exercise 8 (☕, 🪫). The identity functor of a category $\mathbb{1}_C : C \to C$ is a diagram. If it exists, can you describe its limits? And what about its colimit?

Exercise 9 (🪫). Show that a conservative functor preserving equalizers is also faithful.

Leinster 10 (☕, 🪫). 5.3.9

Riehl 11 (🪫). 3.5.i

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adjunctions

Borceux 12 (🔋). 1.11.7

Leinster 13 (☕, 🔋). 5.3.13

Leinster 14 (🔋). 2.1.16

Exercise 15 (🔋). Show that the a right adjoint preserves monomorphisms.

Exercise 16 (🔋). Show that the inclusion of the category of abelian groups in the category of groups $\iota : \mathsf{Ab} \hookrightarrow \mathsf{Grp}$ has a left adjoint.

Exercise 17 (🔋). Prove that the inclusion Haus $\hookrightarrow$ Top of the full subcategory of Hausdorff spaces into the category of all spaces has a left adjoint.

Yoneda

Borceux 18 (🔋). 1.11.11

Leinster 19 (🔋). 4.3.18

Riehl 20 (🔋). 2.2.v

Riehl 21 (🔋). 2.2.vii

- The label **Leinster** refers to the book **Basic Category Theory**, by *Leinster*.
- The label **Borceux** refers to the book **Handbook of Categorical Algebra, Volume I**, by *Borceux*.
- The label **Riehl** refers to the book **Category Theory in context**, by *Riehl*.