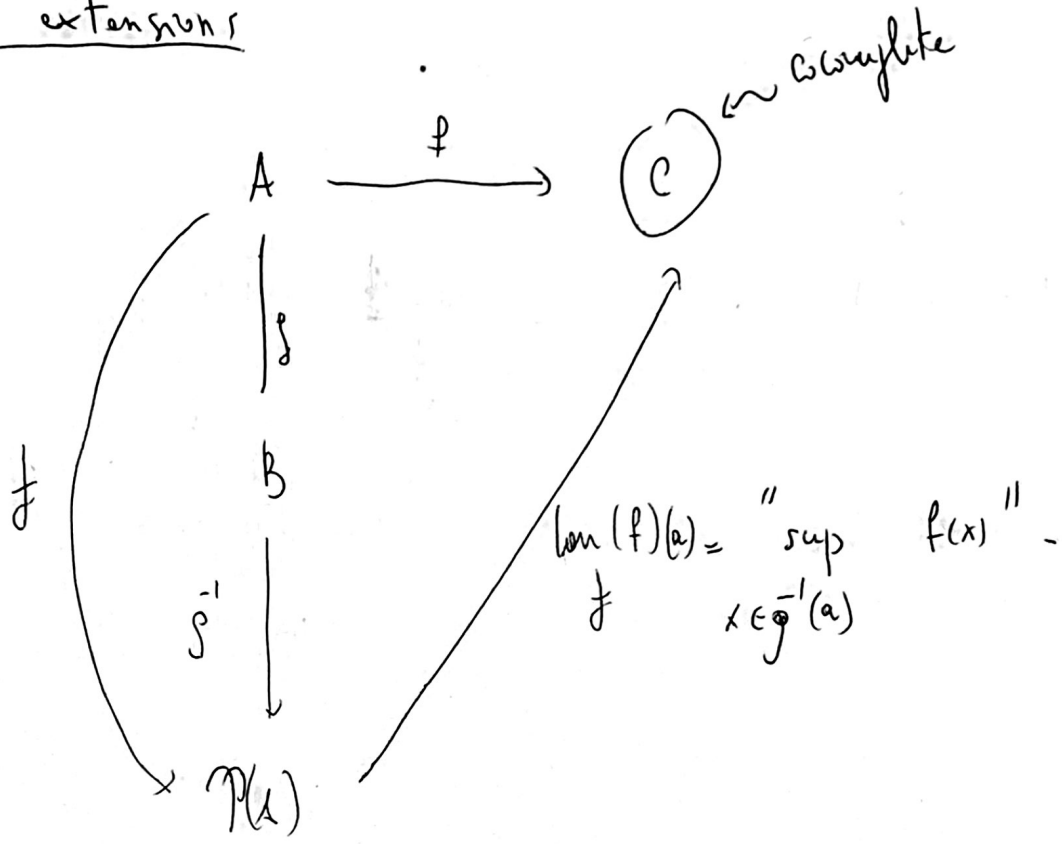


# Kon extensions



Problem! If  $C$  is complete but  $A$  is large. This is cheating.

## Solution



$B(f, -)$  is small.

Define  
arity!!

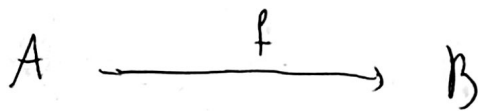
Ex Guide

## Prop

If  $f$  has arity  $A' \subseteq A$ , then the extension exist

$$\text{low}_{g_j} f_j = \text{low}_f (\text{low}_j f_j) = \text{low}_f (f) -$$

AFT

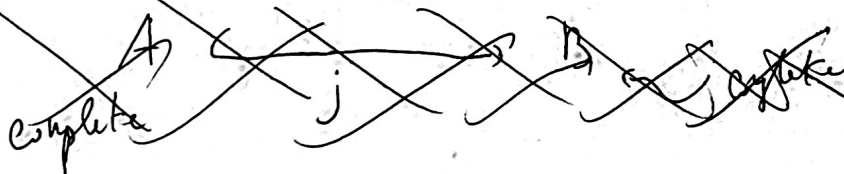


Both complete.  $f$  preserve limits.

$$\text{Then } r = \lim_{\mathcal{I}} 1 = \lim_{\mathcal{I}} \# \circ \boxed{\lim_{\mathcal{I}} f}$$

↑  
Solution set  
condition  
 $B(f-, -)$

~~"Reflexivity"~~



# Lawvere theories

(Highlights)

## (1) Axiomatizing Mon

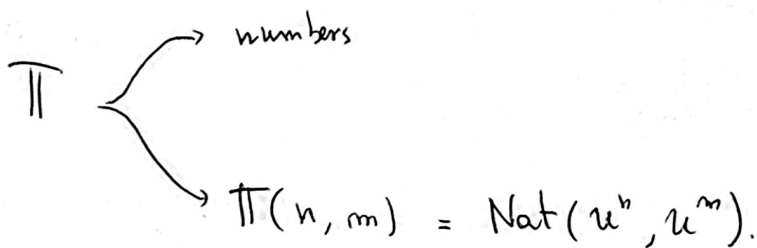
- Lots of implicit operations.

-  $\mathcal{U}: \text{Mon} \rightarrow \text{Set}$

-  $\text{Nat}(\mathcal{U}^n, \mathcal{U}^m)$ .

(\*)  $\text{Nat}(\mathcal{U}^n, \mathcal{U}^m) \cong \text{Mon}(F_m, F_n)$ .

(2)



- (\*)  $\Rightarrow \Pi$  has products.

- Equality between compositions axiomatize  $\left\{ \begin{array}{l} \text{operations} \\ \text{Equations.} \end{array} \right.$

## (3) Functorial semantics

•  $\text{Prod}(\Pi, \text{Set}) \xrightarrow{\kappa} \text{Mon}$  defined in this very special case via  $F(1)$

What about the other direction?

$\mu: F(2) \rightarrow F(1)$   
"  $F(1) \times F(1)$

• Well  $\Pi^{\text{op}} \rightarrow \text{Mon}$ , then nerve.

• Multisorted !!

(4) Properties of  $\text{Mod}(\Pi)$ .

-  $\text{Prod}(\Pi, \text{Set}) \xrightarrow{\quad} \text{Set}^\Pi$   
 closed under limits.

- Has a dense generator.

- it is closed under directed colimits & reflexive coeq.

ARV Mention what are sifted colimits?

"sifted colimits".

Def dir colim / filtered colimit.

Def ref coeq

- Mention Barr-Beck



- $D \rightarrow D^2$  is (co)final.
- (Prop Categories with finite products are complete).

• Commutation of limits and colimits.

• Representables are "tiny".

• It is complete  $\rightarrow$  intro to the s.o.a

$$\begin{array}{ccc}
 f^\#(a \times b) & \xrightarrow{\quad} & F \\
 \downarrow & & \nearrow \\
 f^\#a \times f^\#b & & 
 \end{array}$$

(5) "Interpretation of theories"